

To the applicant:

The following information will help you review math that is included in the Paraprofessional written examination for the Conejo Valley Unified School District. The Education Code requires that no person shall be assigned as an instructional aide/paraprofessional unless the person has demonstrated proficiency in reading, language and mathematics skills up to or exceeding that required by the district for high school seniors.

KEY WORDS AND CONVERTING WORDS TO EQUATIONS

Sometimes math questions use key words to indicate what operation to perform. Becoming familiar with these key words will help you determine what the question is asking for.

OPERATION	Other Words Which Indicate the Operation
Addition	Increased by; more than; combined together; total of; sum; added to. The symbol + means add.
Subtraction	Decreased by; minus; less; difference between/of; less than; fewer than. The symbol – means subtract.
Multiplication	Of; times; multiplied by; product of. The symbols x and · both mean multiply.
Division	Per; a; out of; ratio of; quotient of; percent (divide by 100). The symbol ÷ means divide.
Equal	Is; are; was; will be; gives; yields; sold for. The symbol = means equal.
Per	Divided by
Percent	Divide by 100. The symbol % means percent.

EXPONENTS

An exponent is a superscript or small number written at the top right corner of a number, variable, or parenthesis (for example: 3^3). This tells you to multiply 1 by the number as many times as the exponent says.

SAMPLE	Simplify 3^4
Multiply one by three multiplied by itself three times	$1(3 \times 3 \times 3 \times 3)$
	1 (81)
Answer	81

When multiplying exponents, add the superscripts

SAMPLE	Simplify $x^{16} x^2$
Add the superscripts 16 and 2	x^{16+2}
Answer	x^{18}

When dividing exponents, subtract the superscripts

SAMPLE	Simplify $x^6 \div x^2$
Subtract the superscripts 6 and 2	x^{6-2}
Answer	x^4

FRACTIONS

In order to accurately solve fraction problems it is important to distinguish between the **numerator** and **denominator**. **Numerator** = top number **Denominator** = bottom number

ADDING OR SUBTRACTING FRACTIONS

Adding or subtracting fractions with the **same denominator** is straightforward.

SAMPLE	5/13 + 6/13
The denominator remains the same, add the top numbers	$\frac{5+6}{13}$
Answer	11/13

If you do not have a common denominator, make one by multiplying the first denominator and the second denominator together.

SAMPLE	3/5 + 2/7
Find the common denominator by multiplying five by seven	$5 \times 7 = 35$
To get new numerators, multiply the numerator by the same number as the denominator was multiplied by	$3 \times 7 = 21$ $2 \times 5 = 10$
Insert the new numbers into the numerator and add the fractions	$21/35 + 10/35$
Answer	31/35

MULTIPLYING FRACTIONS

Multiply the numerator times the numerator and the denominator by the denominator.

SAMPLE	1/4 x 3/5
	$\frac{1 \times 3}{4 \times 5}$
Answer	3/20

Simplify the fraction before and after you multiply.

SAMPLE	12/15 x 5/6
Simplify 12/15 by dividing both numbers by 3	$\frac{12 \div 3}{15 \div 3} \times \frac{5}{6}$
	$\frac{4}{5} \times \frac{5}{6}$
Multiply the numerator times the numerator and the denominator by the denominator	$\frac{4 \times 5}{5 \times 6} = \frac{20}{30}$
Simplify by dividing by 10	$\frac{20 \div 10}{30 \div 10}$
Answer	2/3

DIVIDING FRACTIONS

Since division is the opposite of multiplication, first invert one fraction and multiply.

SAMPLE	1/5 ÷ 2/3
Invert 2/3 and multiply	$\frac{1}{5} \times \frac{3}{2}$
Answer	3/10

PRE-ALGEBRA AND ALGEBRA

Special notations for multiplication and division with variables

Here are some examples of special notations and what they mean:

- 2b means $2 \times b$
- $2(a + 5)$ means twice the sum of a number (a) and five
- bc means $b \times c$
- 4bc means $4 \times b \times c$
- $d/5$ means $d \div 5$

Algebra Word Problems

In algebra you solve problems by essentially making two groups, one for each side of an equation. An unknown number or value is represented by a letter (for example: x).

BASIC STEPS

1. Define the variable
2. Translate the problem into an equation and plug in known values
3. Solve the equation
4. Go back to the problem and plug in the new value to obtain the answer

SAMPLE	A car dealership has 15 new cars and 12 used cars. How many cars do they have?
Define the unknown variable	Let $x = \text{Total Cars}$
Translate the problem into an equation and plug known values in	$15 + 12 = x$
Solve the equation	$27 = x$
Answer	There are 27 Total Cars.

SIMPLIFYING EXPRESSIONS

1. **Combine like terms**
2. **Simplify multiplication**
3. **Distribute a number or sign in to parenthesis**
4. **Use the FOIL Method to multiply two or more parenthesis**
5. **Simplify exponents of a number**

1. **Combine like terms** - Combine or add up all of the **like** terms.

Examples of like terms;

- 2x, 45x, x, 0x, -26x because they are all x with a coefficient
- 13, -2, 27, 9043, 0.6 because they are all constants
- $3y^2$, y^2 , $-y^2$, $26y^2$ because they are all y^2 with a coefficient

For comparison, below are a few examples of **unlike** terms:

- 17x, 17z because they are different letter variables
- 15y, $19y^2$, $31y^5$ because they are different powers or exponents
- 19x, 19xy because they both have the letter y but the second term has another variable in it

SAMPLE	$5x + 7x$
Add like terms	$5x + 7x = 12x$
Answer	$12x$

SAMPLE	$14a + 7 + 21a$
Organize like terms together	$14a + 21a + 7$
Add like terms	$14a + 21a + 7 = 35a + 7$
Answer	$35a + 7$

2. Simplify multiplication

Same Variables

SAMPLE	$a \times a$
Neither a has a visible exponent, so their exponents are both 1	$a^1 \times a^1$
Add the exponents	$a^1 \times a^1 = a^2$
Answer	a^2

SAMPLE	$a \times a^4$
The first a doesn't have a visible exponent, so its exponent is 1	$a^1 \times a^4$
Add the exponents	$a^1 \times a^4 = a^5$
Answer	a^5

Different Variables

When multiplying same letter variables, keep the letter and add exponents.

SAMPLE	$y^5 \times a^2$
The terms cannot be multiplied by simply adding the exponents because each multiplier is a different letter	$y^5 \times a^2$
Answer	$y^5 a^2$

SAMPLE	$a^2 \times a^3 y^2$
Add the exponents of a^2 and a^3	$a^5 \times y^2$
Answer	$a^5 y^2$

3. Distribute a number or sign in to parenthesis

SAMPLE	$6(2 + 4a)$
Remove parentheses and multiply each term by six	$(6 \times 2) + (6 \times 4a)$
Answer	$12 + 24a$

4. Use the FOIL Method to multiply two or more parenthesis

- First - Multiply the first term in each parenthesis
- Outer - Multiply the outer term in each parenthesis
- Inner - Multiply the inner term in parenthesis
- Last - Multiply the last term in each parenthesis

SAMPLE	$(3 + 7y)(6 + 2y)$
Multiply the first term	$(3 + 7y)(6 + 2y) = 6 \times 3 = [18]$
Multiply the outer terms	$(3 + 7y)(6 + 2y) = 3 \times 2y = [6y]$ [18 + 6y]
Multiply the inner terms	$(3 + 7y)(6 + 2y) = 7y \times 6 = [42y]$ [18 + 6y + 42y]
Multiply the last terms	$(3 + 7y)(6 + 2y) = 7y \times 2y = [14y^2]$ [18 + 6y + 42y + 14y ²]
Combine like terms	$18 + 6y + 42y + 14y^2$ $18 + 48y + 14y^2$
Answer	$18 + 48y + 14y^2$

5. Simplify Exponents of a number

See the EXPONENTS section for a review

ORDER OF OPERATIONS

1. **Parenthesis and Brackets** from the inside out.
2. **Exponents** of numbers or parenthesis.
3. **Multiplication and Division** in the order they appear.
4. **Addition and Subtraction** in the order they appear

SAMPLE	Simplify the following expression: $2 + (3 - 1)3^2$
Simplify parenthesis and brackets from the inside out (subtract 1 from 3)	$2 + (3 - 1)3^2 = 2 + (2) 3^2$
Simplify exponents (3^2 become 9)	$2 + (2) 3^2 = 2 + 2(9)$
Simplify multiplication and division (multiply 2 and 9)	$2 + 2(9) = 2 + 18$
Simplify addition and subtraction (combine like terms)	$2 + 18 = 20$
Answer	20

PRIME FACTORIZATION

- A **prime number** is a positive integer greater than one that can only be divided by itself and one. Some examples are 2, 3, 5, 7, 11, 13, 17 and 19.
- A **composite number** is a positive integer greater than one that has more than one divisor other than one and itself. Some examples are 4, 6, 9, 15, and 21.
- **One** is neither a prime nor a composite number.

Ways to obtain the prime factor:

- Repeatedly divide by prime numbers
- Choose any pair of factors and split these factors until all the factors are prime.
- Work backwards from the answers, seeing which one is BOTH only prime numbers, and produces the correct product.

SAMPLE	What is the prime factorization for 68?
Divide by 2 (a prime number)	$68 \div 2 = 34$
The correct way to represent prime factorization	2×34
Divide 34 by 2 and you are left with 17 (a prime number)	$34 \div 2 = 17$
Answer	$2 \times 2 \times 17$

FACTORING

Factoring is writing a math expression as a product of factors. For example: writing 14 as (2) (7), where 2 and 7 are factors of 14. Factoring can also be done with trinomial and polynomial expressions.

- **FIRST** – always factor as much as you can. Often all terms in an expression have a common factor, first group the like terms and then find the greatest common factor and extract it.
- Next reverse the FOIL method to get the factored form:
 1. Set up a product of two expressions, where parentheses hold each of the two expressions.
 2. Find the factors that go in the first positions.
 3. Determine the sign for each expression by looking at the signs before the second and third terms in the trinomial:
 - two negative signs: the signs in each expression are opposite with the larger number being negative
 - two positive signs: the signs are both positive
 - negative then a positive: the signs are both negative
 - positive and negative: the signs are opposite and the larger number is positive.
 4. Find the factors that go in the last position.
 5. Check your work

SAMPLE	Factor the trinomial: $x^2 - 4x - 32$
Reverse the FOIL method to get the factored form	$x^2 - 4x - 32$
Since x multiplied by itself equals x^2 , place one x in each parenthesis	$(x +/- ___) (x +/- ___)$
Since the signs on the 2 nd and 3 rd trinomial terms are both negative, the signs of the second term in each factor must be opposite	$(x - ___) (x + ___)$
What two numbers multiplied by one another would equal 32?	Possibilities: 32 and 1 16 and 2 8 and 4
Since the signs on the 2 nd and 3 rd trinomial terms are both negative, the larger multiple will have a negative sign	Possibilities: -32 and 1 -16 and 2 -8 and 4
Out of the possibilities, which pair added to one another equals -4?	-8 and 4
Plug these into the equation	$(x - 8)(x + 4)$
Check your work by using FOIL on the two factors	$X^2 - 8x + 4x - 32$
Simplify	$X^2 - 4x - 32$
Answer	$(x - 8) (x + 4)$

GEOMETRY

Basics

- The angles of any four sided figure always add up to 360° .
- Two lines are **perpendicular** when they meet at a 90° angle.
- Two lines are **parallel** when they never intersect.
- **Bisect** means to cut in half.

Squares

- Each of the 4 sides are always equal in length.
- Each of the 4 angles is always equal to 90° .
- The area (A) of a square is found by squaring the measurement of one side. $A = s^2$.
- Find the perimeter by adding up the length of all the sides. Perimeter = $4s$

Rectangles

- Opposite sides are always equal.
- Each of the 4 angles is always equal to 90° .
- The area of a rectangle is found by multiplying the rectangle's length by its width. $A = lw$
- Find the perimeter by multiplying the length by two and the width by two and adding those products. Perimeter = $2l + 2w$

Circles

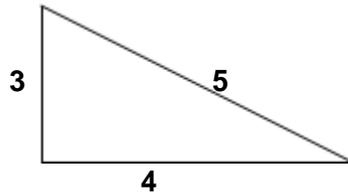
- There are 360° in a circle.
- Radius = distance from the center to any point on the edge of the circle
- $\pi = 3.14$ (B)
- The area of a circle is found by multiplying π by the radius squared. $A = \pi r^2$
- Circumference is the distance around the outside of the circle, find it by multiplying two by π by the radius. Circumference = $2\pi r$ (r).
- Diameter = straight line distance from one point on the circle to another, passing through the center point (d).

Triangles

- Each of the 3 angles will always add up to 180° .
- On right triangles two sides intersect to form a 90° angle.
- The area of a triangle is found by multiplying the triangle's base by its height and dividing the product in half . $A = 1/2bh$
- Find the perimeter by adding up the length of all the sides.
- A hypotenuse is the side of a right triangle that is opposite the right (90°) angle. By using the Pythagorean theorem one can find the length of an unknown side of a right triangle.
- The Pythagorean Theorem is: $a^2 + b^2 = c^2$, where c equals the hypotenuse.

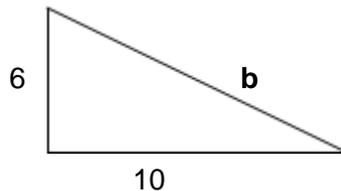
SAMPLE

In the right triangle below, the length of side a = 3, the length of side b = 4 and the hypotenuse (side c) has a length of 5. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$) we see that $3^2 + 4^2 = 5^2$ Simplified to $9+16=25$.



SAMPLE

Determine the length of side **b**, given that side a = 6 and side c = 10



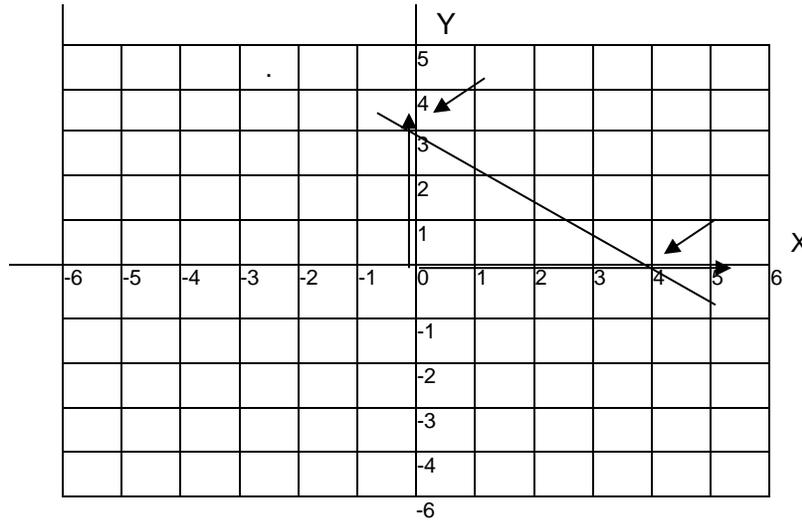
Use the Pythagorean Theorem	$a^2 + b^2 = c^2$
Plug in known values	$6^2 + b^2 = 10^2$
Combine like terms (subtract 6^2 from both sides)	$b^2 = 10^2 - 6^2$
Simplify exponents($1 \times 10 \times 10$) – ($1 \times 6 \times 6$)	$b^2 = 100 - 36$
	$b^2 = 64$
Obtain the square roots $\sqrt{b^2 = 8^2}$	$\sqrt{b^2} = \sqrt{64}$
Answer	$b = 8$

THE COORDINATE SYSTEM

GRID GRAPHS

The location of any point on a grid can be indicated by an **ordered pair** of numbers (X,Y) where X represents the number of units on the horizontal line stemming away from zero (called the x-axis), and Y represents the number of units on the vertical line stemming away from zero (called the y-axis). The X is always listed first, and the Y is always listed second in an ordered pair. The numbers in an ordered pair are called **coordinates**.

For example, if the x-coordinate is 4 and the y-coordinate is 3, the ordered pair for the point would be (4,3).



SLOPE COORDINATES

The **x-intercept** is the point where a line crosses the x-axis. It is found by setting $y = 0$ and solving the resulting equation.

The **y-intercept** is the point where a line crosses the y-axis. It is found by setting $x = 0$ and solving the resulting equation.

SAMPLE	What are the coordinates of the x-intercept of the line $4y - x = 5$?
Set up the equation	$4y - x = 5$
Set $y = 0$ and solve for x	$4(0) - x = 5$
	$-x = 5$
Multiply both sides by -1	$(-1)-x = (-1)5x = -5$
Answer	(-5,0)

SAMPLE	What are the coordinates of the x-intercept of the line $4y - x = 5$?
Set up the equation	$4y - x = 5$
Set $y = 0$ and solve for x	$4(0) - x = 5$
	$-x = 5$
Multiply both sides by -1	$(-1)-x = (-1)5x = -5$
Answer	(-5,0)

MATH GLOSSARY

A

Acute Angle	An angle that is less than a right angle (90°).
Angle	A figure formed when two rays meet at their endpoints. The common endpoint is the vertex.
Area	The number of square units inside of a shape or needed to cover the surface.
Average	A number that is the sum of the terms in a group, divided by the number of terms in that group. An average is also referred to as a mean.

B

Base	In 10^3 , the base is 10. The base is used as a factor as many times as given by the exponent (3). That is $10^3 = 10 \times 10 \times 10$.
Bisect	To divide something in two.

C

Circle Graph	A type of statistical graph used to compare parts of a whole.
Circumference	Distance around a circle.
Coefficient	Any of the factors of a product considered in relation to a specific factor.
Common Multiple	A number that is a multiple of two or more numbers.
Composite Number	A positive integer greater than one that has more than one divisor other than one and itself.
Congruent	Congruent figures match or fit exactly and have the same size and shape.
Consecutive	"in order," usually from smallest to largest.
Constant	A number that is assumed not to change value.
Coordinate of a Point	A number associated with the point on a number line.

D

Decimal	A number with one or more places to the right of the decimal point.
Degree	A unit of measure for an angle.
Denominator	The number below the fraction bar in a fraction.
Diameter	Any line segment that passes through the center of the circle connecting two points on a circle.
Difference	The answer in subtraction.
Distributive Property	For any numbers a, b , and c , $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$
Divisible	A number is divisible by another number if the remainder is 0 after dividing.

E

Equation	An expression with a variable and an equal sign.
Equivalent	Fractions that name the same number. $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{3}{6}$ are equivalent fractions.
Estimate	An approximate rather than an exact answer.
Evaluate	To find the value of an expression by replacing variables with numbers.

Even Number	A number that ends in 0,2,4,6, or 8.
Exponent	In 10^3 , the exponent is 3. The exponent tells how many times the base, 10 is used as a factor. $10^3 = 10 \times 10 \times 10$.
F	
Factor	Numbers or symbols that when multiplied together form a product or that divide another number or symbol.
Formula	An equation that states a rule for the relationship between certain quantities.
Fraction	A number that shows part of a whole unit. Example: $\frac{1}{2}$
G	
Graph	A drawing used to show information.
Greater Than ($>$)	The symbol used to compare two numbers or equations when the greater number is on the left.
Greater Than (\geq)	The symbol used to compare two numbers or equations when the greater or equal number is on the left.
Greatest Common Factor (GCF)	The largest integer that is a common factor of all the given integers.
Grid	Parallel and perpendicular line segments on which points are plotted.
H	
Hundredth	One of 100 equal parts. In the decimal 0.86, the digit 6 is in the hundredths' place.
I	
Integer	All numbers but fractions.
L	
Least Common Denominator (LCD)	The smallest integer that is a common multiple (denominator) of the given integers.
Less Than ($<$)	The symbol used to compare two numbers when the lesser number is on the left.
Less Than (\leq)	The symbol used to compare two numbers when the lesser or equal number is on the left.
Line	A set of points that go on and on in both directions.
Line Graph	A graph that connects points to show how data changes over time.
Line Plot	A graph that shows data along a number line. Each x represents one number in the data set.
Line Segment	Part of a line. It has two endpoints.
M	
Mean	A number that is the sum of the terms in a group, divided by the number of terms in that group. The mean is also known as the average.
Median	The middle number in a set of data when the data are arranged in order from least to greatest.
Mixed Number	A number containing a whole number part and a fraction part, such as $2\frac{1}{2}$.
Mode	The number or numbers that occur most often in a collection of data.

Multiple	The product of a whole number and any other whole number.
Multiplication	An operation between two or more numbers, called factors, to find a product. For example, $5 + 5 + 5$ is the same as 5×3 .

N

Negative Number	Any number less than zero.
Number Line	A line that shows numbers in order.
Numerator	The number above the fraction bar in a fraction.

O

Odd Number	A number that ends in 1,3,5,7, or 9.
Ordered Pair	A pair of numbers that give the location of points on a coordinate grid.

P

Parentheses ()	Grouping symbols. They can show which operation should be done first or multiplication in equations such as $2(3 + 4) = 2 \times 7$.
Percent	A ratio with a denominator of 100. For example 5% and $5/100$ name the same number.
Perimeter	The distance around the outside of a figure.
Perpendicular Lines	Lines that intersect to form right angles.
Pi (B)	The ratio of the circumference of a circle to the diameter of a circle. $B \approx 3.14$
Plot	To locate and mark the point named by an ordered pair on a grid.
Point	An exact location. Points are usually named with capital letters.
Positive Number	Any number greater than zero.
Prime Number	A positive integer greater than one that can only be divided by itself and one.
Probability	The chance that an event will happen.
Product	The answer in multiplication.
Property of One for Division	Any number divided by 1 is that number. Any number except 0 divided by itself is one.
Property of One	The product of any number and 1 is that number. Multiplication

Q

Quotient	The answer in division.
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R

Radius	Any line segment that connects the center of a circle to a point on the circle.
Range	The difference between the greatest number and the least number in a set of data.
Ray	Part of a line. It has one endpoint and continues on and on in one direction.
Rectangle	A quadrilateral with both pairs of opposite sides parallel. There are four right angles.
Remainder	The number left over when division is complete.
Right Angle	An angle that equals 90° .
Rounding	Expressing a number to the nearest ten, hundred, thousand, and so on.

Rules of negative/positive when multiplying or dividing. If there are an even amount of negative signs, the answer is positive and If there are an odd amount of negative signs, the answer is negative.

S

Simplest Form A fraction in which the only common factor of the numerator and denominator is one.

Subtraction To take away one number from another.

Sum The answer in addition.

T

Tenths One or more of ten equal parts of a whole. In the decimal 0.86, the digit 8 is in the tenth's position.

Triangle A polygon with 3 sides and 3 points of intersection.

V

Variable A symbol that stands for a number.

Vertex The point where two rays meet.

Volume The amount of space inside a solid shape.

X

x-axis The horizontal axis in a coordinate grid.

x-coordinate The first number in an ordered pair.

x-intercept The point where the line crosses the x-axis (when $y = 0$ in an equation).

Y

y-axis The vertical axis in a coordinate grid.

y-coordinate The second number in an ordered pair.

y-intercept The point where the line crosses the y-axis (when $x = 0$ in an equation).

Z

Zero Property of Division Zero divided by any number except zero is zero. You cannot divide a number by zero

Zero Property of Multiplication The product of any number and zero is zero.