

Notes 13.4 Probability of Compound Events

I. Independent and Dependent Events

A. Definitions

Probability of Two Independent Events: If two events, A and B, are both independent, then the probability of both events occurring is...

$$P(A \text{ and } B) = P(A) * P(B)$$

Probability of Two Dependent Events: If two events, A and B, are dependent, then the probability of both events occurring is...

$$P(A \text{ and } B) = P(A) * P(B \text{ following } A)$$

B. Examples of Independent and Dependent Events

Example of Independent Events: What is the probability of rolling a 4 on a die 3 times in a row?

$$P(4) = \frac{1}{6}$$

$$P(\text{three 4s}) = \begin{matrix} \text{Roll 1} & \text{Roll 2} & \text{Roll 3} \\ P(4) * & P(4) * & P(4) \end{matrix}$$

$$= \left(\frac{1}{6}\right)^3$$

$$= \frac{1}{216}$$

Example of Dependent Events: What is the probability of drawing 6 hearts from a deck of cards without replacement?

$$\begin{aligned} P(\text{six hearts}) &= \begin{array}{cccccc} \text{Draw 1} & \text{Draw 2} & \text{Draw 3} & \text{Draw 4} & \text{Draw 5} & \text{Draw 6} \\ \frac{13}{52} & * & \frac{12}{51} & * & \frac{11}{50} & * & \frac{10}{49} & * & \frac{9}{48} & * & \frac{8}{47} \end{array} \\ &= \frac{1235520}{14658134400} \\ &= \frac{3861}{4580667} \end{aligned}$$

C. More Examples

Example 1: Suppose the odds of the Sixers beating the Kings in Basketball was 5 : 2. What is the probability of the Sixers beating the Kings 4 times in a row?

$$\text{Odds} = \frac{5}{2} \quad \frac{P(\text{Sixers win})}{P(\text{Kings win})} = \frac{\frac{5}{7}}{\frac{2}{7}} \quad P(\text{Sixers win}) = \frac{5}{7}$$

$$\begin{aligned} P(4 \text{ wins}) &= \begin{array}{cccc} \text{Game 1} & \text{Game 2} & \text{Game 3} & \text{Game 4} \\ \frac{5}{7} & * & \frac{5}{7} & * & \frac{5}{7} & * & \frac{5}{7} \end{array} \\ &= \left(\frac{5}{7}\right)^4 = \frac{625}{2401} \end{aligned}$$

Example 2: A particular bag of marbles contains 4 red, 6 green, 2 blue, and 5 white marbles. What is the probability of picking a red, white, and blue marble, in that order?

$$P(r,w,b) = \begin{array}{c} \textit{Pick 1} \\ \frac{4}{17} \end{array} * \begin{array}{c} \textit{Pick 2} \\ \frac{5}{16} \end{array} * \begin{array}{c} \textit{Pick 3} \\ \frac{2}{15} \end{array} \quad P(r,w,b) = \frac{1}{102}$$

What would the probability be with replacement?

$$P(r,w,b) = \begin{array}{c} \textit{Pick 1} \\ \frac{4}{17} \end{array} * \begin{array}{c} \textit{Pick 2} \\ \frac{5}{17} \end{array} * \begin{array}{c} \textit{Pick 3} \\ \frac{2}{17} \end{array} \quad P(r,w,b) = \frac{40}{4913}$$

II. Mutually Exclusive and Inclusive Events

A. Definitions

Mutually Exclusive Events: If two events, A and B, are mutually exclusive, then that means that if A occurs, then B cannot, and vice versa.

Probability of Mutually Exclusive Events: If two events, A and B, are mutually exclusive, then the probability that either A **OR** B occurs is...

$$P(A \text{ or } B) = P(A) + P(B)$$

Inclusive Events: If two events, A and B, are inclusive, then that means that if A occurs, B could also occur, and vice versa.

Probability of Inclusive Events: If two events, A and B, are inclusive, then the probability that either A or B occurs is...

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

B. Examples

Example 1: A particular bag of marbles contains 4 red, 6 green, 2 blue, and 5 white marbles. If 3 marbles are picked, what is the probability of picking all reds or all greens?

mutually exclusive event

$$P(\text{red or greens}) = P(\text{red}) + P(\text{green})$$

$$= \left[\frac{4}{17} * \frac{3}{16} * \frac{2}{15} \right] + \left[\frac{6}{17} * \frac{5}{16} * \frac{4}{15} \right]$$

$$= \frac{24}{4080} + \frac{120}{4080}$$

$$= \frac{144}{4080} = \frac{3}{85}$$

$$= \frac{C_{(4,3)}}{C_{(17,3)}} + \frac{C_{(6,3)}}{C_{(17,3)}}$$

$$= \frac{4}{680} + \frac{20}{680}$$

$$= \frac{24}{680} = \frac{3}{85}$$

Example 2: Slips of paper numbered 1 to 15 are placed in a box. A slip of paper is drawn at random. What is the probability that the number picked is either a multiple of 5 or an odd number?

inclusive event

$$P(\text{mult of 5 or odd}) = P(\text{mult of 5}) + P(\text{odd}) - P(5 \text{ and odd})$$

$$= \frac{3}{15} + \frac{8}{15} - \frac{2}{15}$$

$$= \frac{9}{15} = \frac{3}{5}$$

Example 3: Two cards are picked out of a standard deck.

What is the probability of both cards being either face cards or clubs?

inclusive event

$$P(\text{face or clubs}) = P(\text{face}) + P(\text{club}) - P(\text{face and club})$$

$$= \left[\frac{12}{52} * \frac{11}{51} \right] + \left[\frac{13}{52} * \frac{12}{51} \right] - \left[\frac{3}{52} * \frac{2}{51} \right]$$

$$= \frac{132}{2652} + \frac{156}{2652} - \frac{6}{2652}$$

$$= \frac{282}{2652} = \frac{47}{442}$$

Ex 4. In a particular group of hospital patients, the probability of having high blood pressure is $\frac{3}{8}$, the probability of having arteriosclerosis is $\frac{5}{12}$, and the probability of having both is $\frac{1}{4}$.

- a) Determine whether the events are mutually exclusive or mutually inclusive.

Mutually inclusive

- a) What is the probability that a patient in this group has either high blood pressure or arteriosclerosis?

$$\frac{3}{8} + \frac{5}{12} - \frac{1}{4} = \frac{13}{24}$$

Probability of Two Independent Events:

$$P(A \text{ and } B) = P(A) * P(B)$$

Probability of Two Dependent Events:

$$P(A \text{ and } B) = P(A) * P(B \text{ following } A)$$

Probability of Mutually Exclusive Events:

$$P(A \text{ or } B) = P(A) + P(B)$$

Probability of Inclusive Events:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$