I. Sequences and Terms

- **Sequence**: a list of numbers in a specific order.

  \[1, 3, 4, 7, 10, 16\]

- **Term**: each number in a sequence
II. Arithmetic Sequences

A. Definition

A sequence is **arithmetic** if the differences between consecutive terms are the same.

\[ \begin{align*}
9 - 4 &= 5 \\
14 - 9 &= 5 \\
19 - 14 &= 5 \\
24 - 19 &= 5
\end{align*} \]

The **common difference**, \( d \), is 5.

FYI: Common differences can be negative.
How do I know if it is an arithmetic sequence?

• Look for a common *difference* between consecutive terms

Ex: 2, 4, 8, 16 . . .

Common Difference?

2 + 2 = 4
4 + 4 = 8
8 + 8 = 16

No. The sequence is NOT Arithmetic

Ex: 48, 45, 42, 39 . . .

Common Difference?

48 - 3 = 45
45 - 3 = 42
42 - 3 = 39

Yes. The sequence IS Arithmetic. $d = -3$
III. Finding Subsequent Terms

- Find the next three terms in the arithmetic sequence:
  2, 5, 8, 11, 14, __, __, __
  2, 5, 8, 11, 14, 17, 20, 23
- The common difference is?
- 3!!!
IV. Finding the $n^{th}$ Term of an Arithmetic Sequence

$$a_n = a_1 + (n - 1)d$$

Where:

- $a_n$ is the $n$th term in the sequence
- $a_1$ is the first term
- $n$ is the number of the term
- $d$ is the common difference
Ex 1: Find the 25\textsuperscript{th} term in the sequence of 5, 11, 17, 23, 29 \ldots

Common difference
\[ a_2 - a_1 = 11 - 5 = 6 \]

\[ a_n = a_1 + (n - 1)d \]

\[ a_{25} = 5 + (25 - 1)6 \]

\[ a_{25} = 5 + (24)6 = 149 \]
Ex 2: Find the 17\textsuperscript{th} term of the arithmetic sequence: 26, 13, 0, -13

Common difference

\[ a_2 - a_1 = 13 - 26 = -13 \]

\[ a_n = a_1 + (n-1)d \]

Find the common difference between the values.

Start with the explicit sequence formula

\[ a_{25} = 26 + (17 -1)(-13) \]

Plug in known values

\[ a_{25} = 26 + (16)(-13) = -182 \]

Simplify
Ex 3: Find the first term of an arithmetic sequence if the 9th term is 72 and the common difference is 5.

\[ a_n = a_1 + (n-1)d \]

\[ 72 = a_1 + (9-1)5 \]

\[ 72 = a_1 + (8)5 \]

\[ 72 = a_1 + 40 \]

\[ 32 = a_1 \]
Ex. 4: Suppose you have saved $75 towards the purchase of a new tablet. You plan to save at least $12 from mowing your neighbor’s yard each week. In all, what is the minimum amount of money you will have in 26 weeks?

\[
a_n = a_1 + (n-1)d
\]

Find the common difference between the values. You will save $12 a week so this is your difference.

\[
a_{26} = 75 + (27 -1)12
\]

Start with the explicit sequence formula

Substitute in known values

WAIT: Why 27 and not 26 for n?

The first term in the sequence, 75, came before the weeks started (think of it as week 0). Therefore you want one more week in your formula to account for the $75 that you had before you started saving.

\[
a_{26} = 75 + (26)12 = $387
\]

Simplify
VI. Arithmetic Means

Arithmetic Means: the terms between any two nonconsecutive terms of an arithmetic sequence.

Example: 17, 10, 3, -4, -11, -18, ...

Between 10 and -18 there are three arithmetic means 3, -4, -11.
Ex 2: Find three arithmetic means between 8 and 14.

- So our sequence must look like $8, __, __, __, 14$.
- In order to find the means we need to know the common difference. We can use our formula to find it.

\[8, __, __, __, 14\]
\[a_1 = 8, a_5 = 14, \quad n = 5\]

\[14 = 8 + d(5 - 1)\]
\[14 = 8 + d(4)\]
\[6 = 4d\]
\[1.5 = d\]

\[8, __, __, __, 14\] so to find our means we just add 1.5 starting with 8.

\[8, 9.5, 11, 12.5, 14\]
VII. Arithmetic Series

- A series is the expression for the **sum of the terms of a sequence**, not just “what is the next term?”

  Ex: 6, 9, 12, 15, 18 . . . 
  
  This is a list of the numbers in the pattern an not a sum. It is a sequence. Note it goes on forever, so we say it is an infinite sequence.

Ex: 6 + 9 + 12 + 15 + 18

  Here we are adding the values. We call this a series. Because it does not go on forever, we say it is a **finite** series.

Note: if the numbers go on forever, it is infinite; if it has a definitive ending it is finite.
Sum of a Finite Arithmetic Series

\[ S_n = \frac{n}{2} (a_1 + a_n) \]

Where: \( S_n \) is the sum of all the terms

\( n \) = number of terms

\( a_1 \) = first term

\( a_n \) = last term

For Example, \( 2 + 11 + 20 + 29 + 38 + 47 = 147 \)

\[ S_n = \frac{6}{2} (2 + 47) = 147 \]

Let’s try one: evaluate the series: 5, 9, 13, 17, 21, 25, 29

\[ S_n = \frac{7}{2} (5 + 29) = 119 \]
VIII. Application of Arithmetic Series

• Ofelia sells houses in a new development. She makes a commission of $3750 on the sale of her first house. For each additional house sold, her commission increases by $500. Thus on her next house she will make $4250. How many houses will she have to sell for her total commission to be at least $65,000?

• In this situation, her commission is increasing by the same amount each time, and we are asking for the sum of all her commissions. Therefore, this represents an arithmetic series. We are solving for n.
Because we have two unknowns (n and $a_n$), we need to substitute something in for one of them. We can substitute

- $a_1 + (n-1)d$ for $a_n$

\[
S_n = \frac{n}{2} (a_1 + a_n)
\]

\[
S_n = \frac{n}{2} (a_1 + (a_1 + (n-1)d))
\]

\[
S_n = \frac{n}{2} (2a_1 + (n-1)d)
\]

\[
65000 = \frac{n}{2} (2(3750) + (n-1)(500))
\]

\[
65000 = \frac{n}{2} (7500 + 500n - 500)
\]
\[65000 = \frac{n}{2} (7000 + 500n)\]

\[130000 = 7000n + 500n^2\]

\[0 = 500n^2 + 7000n - 130000\]

\[0 = n^2 + 14n - 260\]

\[n = 10.58; n = -24.58\]

She will need to sell 11 houses to make at least $65,000 a year.