Notes: Surface Area of Right Prisms and Cylinders
A. Definitions and Vocabulary

Prisms and cylinders have 2 congruent parallel bases. A **lateral face** is not a base. The edges of the base are called **base edges**. A **lateral edge** is not an edge of a base. The lateral faces of a **right prism** are all rectangles.

![Diagram of a right prism with labels for bases, base edges, lateral edges, and lateral faces.](image-url)
An **altitude** of a prism or cylinder is a perpendicular segment joining the planes of the bases. The *height* of a three-dimensional figure is the length of an altitude. For a right prism, the altitude is the same as the length of a lateral side.

**Surface area** is the total area of all faces and curved surfaces of a three-dimensional figure. The *lateral area* of a prism is the sum of the areas of the lateral faces.
The net of a right prism can be drawn so that the lateral faces form a rectangle with the same height as the prism. The base of the rectangle is equal to the perimeter of the base of the prism.

\[ P = a + b + c \]
The surface area of a right rectangular prism with length \( \ell \), width \( w \), and height \( h \) can be written as \( S = 2\ell w + 2wh + 2\ell h \).
Ex 1a. Find the lateral area and surface area of the right rectangular prism. Round to the nearest tenth, if necessary.

L = Ph
= 32(14) = 448 ft

P = 2(9) + 2(7) = 32 ft

S = Ph + 2B
= 448 + 2(7)(9) = 574 ft²
Ex 1b. Find the lateral area and surface area of a right regular triangular prism with height 20 cm, base edges of length 10 cm, and base area of $25\sqrt{3}$ cm$^2$. Round to the nearest tenth, if necessary.

$L = Ph$
$= 30(20) = 600$ ft$^2$

$S = Ph + 2B$
$= 600 + 50\sqrt{3} \approx 686.6$ cm$^2$
Ex 1c. Find the lateral area and surface area of a cube with edge length 8 cm.

\[ L = Ph \]
\[ = 32(8) = 256 \text{ cm}^2 \]

\[ S = Ph + 2B \]
\[ = 256 + 2(8)(8) = 384 \text{ cm}^2 \]

\[ P = 4(8) = 32 \text{ cm} \]
III. Right Cylinders

The **lateral surface** of a cylinder is the curved surface that connects the two bases. The **axis of a cylinder** is the segment with endpoints at the centers of the bases. The axis of a **right cylinder** is perpendicular to its bases. The altitude of a right cylinder is the same length as the axis.
The lateral area of a right cylinder with radius \( r \) and height \( h \) is \( L = 2\pi rh \).

The surface area of a right cylinder with lateral area \( L \) and base area \( B \) is \( S = L + 2B \), or \( S = 2\pi rh + 2\pi r^2 \).
Ex 2a. Find the lateral area and surface area of the right cylinder. Give your answers in terms of $\pi$.

The radius is half the diameter, or 8 ft.

$L = 2\pi rh = 2\pi(8)(10) = 160\pi \text{ in}^2$

$S = L + 2\pi r^2 = 160\pi + 2\pi(8)^2$

$= 288\pi \text{ in}^2$
Ex 2b: Find the lateral area and surface area of a right cylinder with circumference $24\pi$ cm and a height equal to half the radius. Give your answers in terms of $\pi$.

**Step 1** Use the circumference to find the radius.

\[
C = 2\pi r
\]

Circumference of a circle

\[
24\pi = 2\pi r
\]

Substitute $24\pi$ for $C$.

\[
r = 12
\]

Divide both sides by $2\pi$. 
Ex 2c. Find the lateral area and surface area of a right cylinder with circumference $24\pi$ cm and a height equal to half the radius. Give your answers in terms of $\pi$.

**Step 2** Use the radius to find the lateral area and surface area. The height is half the radius, or 6 cm.

\[ L = 2\pi rh = 2\pi(12)(6) = 144\pi \text{ cm}^2 \quad \text{Lateral area} \]

\[ S = L + 2\pi r^2 = 144\pi + 2\pi(12)^2 \]
\[ = 432\pi \text{ in}^2 \quad \text{Surface area} \]
Ex 2d. Find the lateral area and surface area of a cylinder with a base area of $49\pi$ and a height that is 2 times the radius.

**Step 1** Use the circumference to find the radius.

\[ A = \pi r^2 \]  
\[ 49\pi = \pi r^2 \]  
\[ r = 7 \]  

*Area of a circle*  
*Substitute $49\pi$ for $A$.*  
*Divide both sides by $\pi$ and take the square root.*
Ex 2e. Find the lateral area and surface area of a cylinder with a base area of $49\pi$ and a height that is 2 times the radius.

**Step 2** Use the radius to find the lateral area and surface area. The height is twice the radius, or 14 cm.

$L = 2\pi rh = 2\pi(7)(14)=196\pi \text{ in}^2$  \hspace{1cm} \text{Lateral area}

$S = L + 2\pi r^2 = 196\pi + 2\pi(7)^2 =294\pi \text{ in}^2$  \hspace{1cm} \text{Surface area}