Notes: Area of Quadrilaterals and Triangles
I. Area of a Rectangles and Parallelogram

Recall that a rectangle with base $b$ and height $h$ has an area of $A = bh$.

You can see from below that a parallelogram has the same area as a rectangle with the same base and height.

A triangle is cut off one side and translated to the other side.
The area of a parallelogram with base \( b \) and height \( h \) is \( A = bh \).

The height of a parallelogram is measured along a segment perpendicular to a line containing the base.

**Remember!**

The perimeter of a rectangle with base \( b \) and height \( h \) is \( P = 2b + 2h \) or 
\[
P = 2 (b + h).
\]
Ex 1a: Find the area of the parallelogram.

**SOLUTION**

Use the formula for the area of a parallelogram. Substitute 9 for $b$ and 6 for $h$.

$$A = bh$$  \hspace{1cm} \text{Formula for the area of a parallelogram}

$$= (9)(6)$$  \hspace{1cm} \text{Substitute 9 for $b$ and 6 for $h$.}

$$= 54$$  \hspace{1cm} \text{Multiply.}

**ANSWER**

The parallelogram has an area of 54 square meters.
Ex 1b: Find the base of the parallelogram in which $h = 56 \text{ yd}$ and $A = 28 \text{ yd}^2$.

\[
A = bh
\]

\[
28 = b(56)
\]

\[
\begin{array}{c|c}
56 & 56 \\
\end{array}
\]

\[
b = 0.5 \text{ yd}
\]
Ex 1c: Find the height of a rectangle in which \( b = 3 \) in. and \( A = (6x^2 + 24x - 6) \) in\(^2\).

\[
A = bh
\]

Area of a rectangle

\[
6x^2 + 24x - 6 = 3h
\]

Substitute \( 6x^2 + 24x - 6 \) for \( A \) and 3 for \( b \).

\[
3(2x^2 + 8x - 2) = 3h
\]

Factor 3 out of the expression for \( A \).

\[
2x^2 + 8x - 2 = h
\]

Divide both sides by 3.

\[
h = (2x^2 + 8x - 2) \text{ in.}
\]
II. Triangles and Trapezoids

The area of a triangle with base $b$ and height $h$ is $A = \frac{1}{2}bh$.

The area of a trapezoid with bases $b_1$ and $b_2$ and height $h$ is $A = \frac{1}{2}(b_1 + b_2)h$, or $A = \frac{(b_1 + b_2)h}{2}$.
Ex 2a: Find the area of a trapezoid in which $b_1 = 8$ in., $b_2 = 5$ in., and $h = 6.2$ in.

$$A = \frac{1}{2} (b_1 + b_2) h$$  \hspace{1cm} \text{Area of a trapezoid}

$$A = \frac{1}{2} (8 + 5)(6.2)$$ \hspace{1cm} \text{Substitute 8 for $b_1$, 5 for $b_2$, and 6.2 for $h$.}

$$A = 40.3 \text{ in}^2$$ \hspace{1cm} \text{Simplify.}
Ex 2b: Find the base of the triangle, in which $A = (15x^2) \text{ cm}^2$.

$$A = \frac{1}{2} bh$$

Area of a triangle

$$15x^2 = \frac{1}{2} b(5x)$$

Substitute $15x^2$ for $A$ and $5x$ for $h$.

$$15x = \frac{5}{2} b$$

Divide both sides by $x$.

$$6x = b$$

Multiply both sides by $\frac{2}{5}$.

$$b = 6x \text{ cm}$$

Sym. Prop. of =
Ex 2c: Find $b_2$ of the trapezoid, in which $A = 231 \text{ mm}^2$.

\[ A = \frac{1}{2}(b_1 + b_2)h \]

\[ 231 = \frac{1}{2}(23 + b_2)(11) \]

Multiply both sides by \(\frac{2}{11}\).

\[ 42 = 23 + b_2 \]

Subtract 23 from both sides.

\[ 19 = b_2 \]

\[ b_2 = 19 \text{ mm} \]
III. Areas of Rhombi

The area of a rhombus with diagonals $d_1$ and $d_2$ is $A = \frac{1}{2} d_1 d_2$
Ex 3a: Find $d_2$ of a rhombus in which $d_1 = 14$ in. and $A = 238$ in$^2$.

\[ A = \frac{1}{2} d_1 d_2 \quad \text{Area of a rhombus} \]

\[ 238 = \frac{1}{2} (14) d_2 \quad \text{Substitute 238 for } A \text{ and } 14 \text{ for } d_1. \]

\[ 34 = d_2 \quad \text{Solve for } d_2. \]

\[ d_2 = 34 \quad \text{Sym. Prop. of } = \]
Ex 3b: Find the area of a rhombus.

\[ A = \frac{1}{2} d_1 d_2 \]  
\text{Area of a rhombus}

\[ A = \frac{1}{2} (8x + 7)(14x - 6) \]  
\text{Substitute (8x+7) for } d_1 \text{ and (14x-6) for } d_2. \]

\[ A = \frac{1}{2} (112x^2 + 50x - 42) \]  
\text{Multiply the binomials (FOIL).}

\[ A = (56x^2 + 25x - 21) \text{ cm}^2 \]  
\text{Distrib. Prop.}
Ex 3c: Find $d_2$ of a rhombus in which $d_1 = 3x \text{ m}$ and $A = 12xy \text{ m}^2$.

\[ A = \frac{1}{2} d_1 d_2 \quad \text{Formula for area of a rhombus} \]

\[ 12xy \text{ m}^2 = \frac{1}{2} (3x \text{ m}) (d_2) \quad \text{Substitute.} \]

\[ d_2 = 8y \text{ m} \quad \text{Simplify.} \]