Variables and Expressions,
Order of Operations

Notes 1.1 and 1.2
I. Expressions

You are familiar with the following type of numerical expressions:

- $12 + 6$
- $3 (12)$
- $6 (3 + 2)$
- $15 - 4 (6)$
II. Variables

In the expression 12 + B, the letter “B” is a variable.

**Definition:**

A **variable** is a letter or symbol that represents an unknown value.
III. Algebraic Expressions

When variables are used with other numbers, parentheses, or operations, they create an algebraic expression.

\[
\begin{align*}
a + 2 \\
(a) (b) \\
3m + 6n - 6
\end{align*}
\]
## Expressions vs. Equations

<table>
<thead>
<tr>
<th>Expressions</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No = sign</td>
<td>= sign</td>
</tr>
<tr>
<td>Simplify</td>
<td>Solve</td>
</tr>
</tbody>
</table>
IV. Coefficients

A **coefficient** is the number multiplied by the variable in an algebraic expression.

<table>
<thead>
<tr>
<th>Algebraic Expression</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>6m + 5</td>
<td>6</td>
</tr>
<tr>
<td>8r + 7m + 4</td>
<td>8, 7</td>
</tr>
<tr>
<td>14b - 8</td>
<td>14</td>
</tr>
</tbody>
</table>
IV. Terms

A **term** is the name given to a number, a variable, or a number and a variable combined by multiplication or division.

<table>
<thead>
<tr>
<th>Algebraic Expressions</th>
<th>Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a + 2$</td>
<td>$a, 2$</td>
</tr>
<tr>
<td>$3m + 6n - 6$</td>
<td>$3m, 6n, - 6$</td>
</tr>
</tbody>
</table>
V. Constants

- A constant is a number that cannot change its value.

In the expression: $5x + 7y + 2$
the constant is $2$

In the expression: $x - 3$
the constant is $-3$
VI. Factors and Products

$(3)(5) = 15$

- Factors: Quantities being multiplied
- Products: The result of multiplying factors

These are **factors**

This is a **product**
VII. Powers, Bases and Exponents

This is a **Power**

This is a **base**

This is an **exponent**
Let’s Practice!

Identify the terms, coefficients, and constants.

<table>
<thead>
<tr>
<th>Expression</th>
<th>TERMS</th>
<th>COEFFICIENTS</th>
<th>CONSTANTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>12a – 6b + 4</td>
<td>12a, -6b, 4</td>
<td>12, -6</td>
<td>4</td>
</tr>
<tr>
<td>4x – 2y</td>
<td>4x, -2y</td>
<td>4, -2</td>
<td>0</td>
</tr>
<tr>
<td>C - 32</td>
<td>C, -32</td>
<td>1</td>
<td>-32</td>
</tr>
<tr>
<td>3x + 2</td>
<td>3x, 2</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
VIII. Writing Algebraic Expressions

• A. You can translate word phrases into variable expressions.
  – Examples:
  1. Three more than a number = $x + 3$
  2. The quotient of a number and 8 = $y/8$
  3. Six times a number = $6 \cdot n$ or $6n$
  4. 15 less than a number = $z - 15$
  5. The quotient of 30 and a number plus 10 = $30/x + 10$. 
B. Key words to look for:

• Addition:
  – Add
  – Plus
  – Sum
  – Total
  – Increased by
  – More than

• Subtraction:
  – Minus
  – Difference
  – Subtract
  – Less than
  – Decreased by
  – less
Cont...

- Multiplication
  - Product
  - Times
  - Multiply
  - per

- Division
  - Quotient
  - Divide
  - Split Between
C. Write algebraic expressions for these word phrases

1. Four more than s
2. The product of 7 and c
3. Nine less than x
4. A number divided by the sum of 4 and 7.
5. Twice the sum of a number plus 4.
6. The sum of \( \frac{3}{4} \) of a number and 7.
7. Ten times a number increased by 150.
D. Write an algebraic phrase for these situations

1. A car was traveling 35 miles per hour for a number of hours.
2. Bob ran 7 times a week for a number of weeks.
3. The plumber added an extra $35 to her bill.
4. Thirty-five fewer people came than the number expected.
IX. ORDER OF OPERATIONS

When an expression contains more than one operation, the order of operations tells you which operation to perform first.
<table>
<thead>
<tr>
<th></th>
<th>Order of Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td><strong>Grouping:</strong> Perform operations inside grouping symbols.</td>
</tr>
<tr>
<td>2.</td>
<td><strong>Powers:</strong> Evaluate all powers.</td>
</tr>
<tr>
<td>3.</td>
<td><strong>Mult/Div:</strong> Multiply and/or divide in order from left to right.</td>
</tr>
<tr>
<td>4.</td>
<td><strong>Add/Sub:</strong> Add and/or subtract from left to right.</td>
</tr>
</tbody>
</table>
Grouping symbols include parentheses ( ), brackets [ ], and braces { }. If an expression contains more than one set of grouping symbols, begin with the innermost set. Follow the order of operations within that set of grouping symbols and then work outward.

**Helpful Hint**

Fraction bars, radical symbols, and absolute-value symbols can also be used as grouping symbols. Remember that a fraction bar indicates division.
Ex 1: Simplifying Numerical Expressions

Simplify each expression.

A. \( 15 - 2 \cdot 3 + 1 \)

\[
\begin{align*}
15 & - 2 \cdot 3 + 1 \\
15 & - 6 + 1 \\
9 & + 1 \\
10 &
\end{align*}
\]

There are no grouping symbols.

Multiply.

Subtract.

Add.

B. \( 12 + 3^2 + 10 \div 2 \)

\[
\begin{align*}
12 & + 3^2 + 10 \div 2 \\
12 & + 9 + 10 \div 2 \\
12 & + 9 + 5 \\
26 &
\end{align*}
\]

There are no grouping symbols.

Evaluate powers. The exponent applies only to the 3.

Divide.

Add.
Simplify each expression.

c. \[ \frac{2(-4) + 22}{4^2 - 9} \]

\[ \frac{2(-4) + 22}{16 - 9} \]

\[ \frac{-8 + 22}{7} \]

\[ \frac{14}{7} = 2 \]

The fraction bar is a grouping symbol.

Evaluate powers. The exponent applies only to the 4.

Multiply above the bar and subtract below the bar.

Add above the bar and then divide.
Simplify the expression.

D. \[ 8 \div \frac{1}{2} \cdot 3 \]

\[ 8 \div \frac{1}{2} \cdot 3 \]

\[ \frac{8}{1} \cdot \frac{2}{1} \cdot 3 \]

\[ 16 \cdot 3 \]

\[ 48 \]

*There are no grouping symbols.*

*Rewrite division as multiplication.*

*Multiply.*
E. Simplify the expression.

\[ 3\sqrt{50} - 1 \]

\[ 3\sqrt{49} \quad \text{The square root sign acts as a grouping symbol.} \]

\[ 3 \cdot 7 \quad \text{Subtract.} \]

\[ 21 \quad \text{Take the square root.} \]

\[ \text{Multiply.} \]
F. Simplify the expression.

\[
\frac{(5 + 2)(-8)}{(-2)^3 - 3} \quad \text{The division bar acts as a grouping symbol.}
\]

\[
\frac{7(-8)}{-8 - 3} \quad \text{Add and evaluate the power.}
\]

\[\frac{-56}{-11} = 5 \frac{1}{11} \quad \text{Multiply, subtract and simplify.}\]
X. Evaluating Algebraic Expressions

Evaluate $a^2 - (b^3-4c)$ if $a = 7$, $b = 3$, and $c = 5$.

- To evaluate an algebraic expression, replace the variables with their values. Then, find the value of the numerical expression using order of operations.
A. Evaluate $a^2 - (b^3 - 4c)$ if $a = 7$, $b = 3$, and $c = 5$.

$$a^2 - (b^3 - 4c) = 7^2 - (3^3 - 4 \cdot 5)$$

Replace $a$ with 7, $b$ with 3, and $c$ with 5.

$$= 49 - (27 - 4 \cdot 5)$$

Evaluate $7^2$ and $3^3$

$$= 49 - (27 - 20)$$

Multiply 4 and 5

$$= 49 - 7$$

Subtract 20 from 27

$$= 42$$

Subtract
B. • Evaluate \( x(y^3 + 8) \div 12 \) if \( x = 3 \), and \( y = 4 \).

\[
x(y^3 + 8) \div 12 = 3(4^3 + 8) \div 12
\]

Replace \( x \) with 3 and \( y \) with 4.

\[
= 3(64 + 8) \div 12
\]

Evaluate \( 4^3 \)

\[
= 3(72) \div 12
\]

Add 64 and 8

\[
216 \div 12
\]

Multiply 72 and 3

\[
= 18
\]

Divide
Business Connection: Consumerism

c. According to market research, the average consumer spends $78 per trip to the mall on the weekends and only $67 per trip during the week.

--Write an algebraic expression to represent how much the average consumer spends at the mall in x weekend trips and y weekday trips.

$78x + 67y$

--Evaluate the expression to find what the average consumer spends after going to the mall twice during the week and 5 times on the weekends.

$78(5) + 67(2) = 390 + 134 = 554$

The average consumer spends $554.